American University of Beirut

MATH 201

Calculus and Analytic Geometry III

Fall 2006-2007

quiz # 1

Name: .... ID #: ....

1. *(15 points, 5 points each)* Find each of the following limits:

a) $\lim\limits_{n\to+\infty} (-1)^n \dfrac{n}{n+1} + 1$

b) $\lim\limits_{n\to+\infty} \dfrac{\ln^{10} n}{n + 10\, n^{2/n}}$

c) $\lim\limits_{n\to+\infty} \left(\dfrac{3n+1}{3n}\right)^{2n-1}$

2. *(10 points)* Show that the sequence $a_n = \dfrac{(n!)^2\, 4^n}{(2n)!}$ is nondecreasing.

What can you say about the series $\displaystyle\sum_{n=0}^{+\infty} \frac{(n!)^2\, 4^n}{(2n)!}$? justify.

3. *(30 points, 10 points each)* Determine if the following **series** converges or diverges. **Justify your answers**

a) $\displaystyle\sum_{n=0}^{+\infty} \frac{\ln n + e^n}{2^{2n} + n^3}$

b) $\displaystyle\sum_{n=1}^{+\infty} \left(\frac{e}{\left(1+\frac{1}{2n}\right)^n}\right)^n$

c) $\displaystyle\sum_{n=2}^{+\infty} \frac{1}{n^{\ln n}}$

4. *(15 points)* Find $\displaystyle\sum_{n=0}^{\infty} \left[(-1)^n \frac{\pi^{2n+1}}{4^n (2n+1)!} + \frac{2^{3n}}{3^{2n+2}}\right]$

5. **a)** *(18 points)* Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{+\infty} \frac{(x+1)^n}{1+\ln n}$

(*be sure to check convergence at the endpoints*)

b) *(2 points)* For what value(s) of x for which the series converges (i) absolutely? (ii) conditionally?

6. (10 points) Show that for $x > 0$, $1 - \dfrac{x}{2} < \dfrac{\ln(1+x)}{x} < 1$.

(*hint: use the Maclaurin series of* $\ln(1+x)$ *and the Alternating Estimation Theorem*)